

Math 270 Linear Algebra

Chapter 3 Determinants

3.5 Cramer's Rule

Let $A\vec{x} = \vec{b}$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

If $\det(A) \neq 0$, then

$$x_1 = \frac{\det(A_1)}{\det(A)}, x_2 = \frac{\det(A_2)}{\det(A)}, \dots, x_n = \frac{\det(A_n)}{\det(A)}$$

where A_i is the matrix obtained from A by replacing the i th column of A by \vec{b} .

Exercise #4

$$2x_1 + x_2 + x_3 = 6$$

$$3x_1 + 2x_2 - 2x_3 = -2$$

$$x_1 + x_2 + 2x_3 = -4$$

Solution:

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & -2 \\ 1 & 1 & 2 \end{vmatrix} \stackrel{1\text{st row}}{=} 2(4+2) - 1(6+2) + 1(3-2) = 12 - 8 + 1 = 5$$

$$|A_1| = \begin{vmatrix} 6 & 1 & 1 \\ -2 & 2 & -2 \\ -4 & 1 & 2 \end{vmatrix} \stackrel{1\text{st row}}{=} 6(4+2) - 1(-4-8) + 1(-2+8) = 36 + 12 + 6 = 54$$

$$|A_2| = \begin{vmatrix} 2 & 6 & 1 \\ 3 & -2 & -2 \\ 1 & -4 & 2 \end{vmatrix} \stackrel{1\text{st row}}{=} 2(-4-8) - 6(6+2) + 1(-12+2) = -24 - 48 - 10 = -82$$

$$|A_3| = \begin{vmatrix} 2 & 1 & 6 \\ 3 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix} \stackrel{1\text{st row}}{=} 2(-8+2) - 1(-12+2) + 6(3-2) = -12 + 10 + 6 = 4$$

$$x_1 = \frac{54}{5}, \quad x_2 = -\frac{82}{5}, \quad x_3 = \frac{4}{5}$$