Math 270 Linear Algebra

Chapter 3 Determinants

3.1 Definition

Given $S = \{1, 2, ..., n\}$.

permutation of S

ex. $S = \{1, 2, 3\}$

213	231
321	312
132	123

inversion in a permutation

213	one inversion
321	three inversions
123	no inversions

even permutation — even number of inversions
odd permutation — odd number of inversions

S_n = set of all permutations of S

Ex. $S_3 = \{123,\ 231,\ 312,\ 132,\ 321,\ 213\}$ has $6 = 3!$ elements

Definition Let $A = \left[a_{ij}\right]$ be an n x n matrix. The **determinant** of A,

$$\det A = \sum(\pm)\ a_{1j_1}a_{2j_2}...a_{nj_n},$$

where the Σ is taken over all permutations $j_1j_2...j_n$ of the set $S = \{1, 2, ..., n\}$. The + or – depends on whether the permutation is even or odd.

Example 1 $A = [a_{11}]$

$\det A = |A| = a_{11}$

Example 2

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \Rightarrow +a_{1\underline{1}}a_{2\underline{2}} - a_{1\underline{2}}a_{2\underline{1}}$$

$$|A| = a_{11}a_{12} - a_{12}a_{21}$$

a_{11} a_{12}

a_{21} a_{22}

$$ex: A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow |A| = (1)(4) - (3)(4) = 4 - 6 = -2$$

Example 3 $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a33 \end{bmatrix}$ $A = +a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 2 \\ 3 & -1 & 2 \end{bmatrix} \begin{matrix} 1 & 1 \\ 2 & 1 \\ 3 & -1 \end{matrix}$$